Geometric patterns and 3D objects: An Intermediate Phase training manual.

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IP training manual: Geometric patterns and 3-D objects

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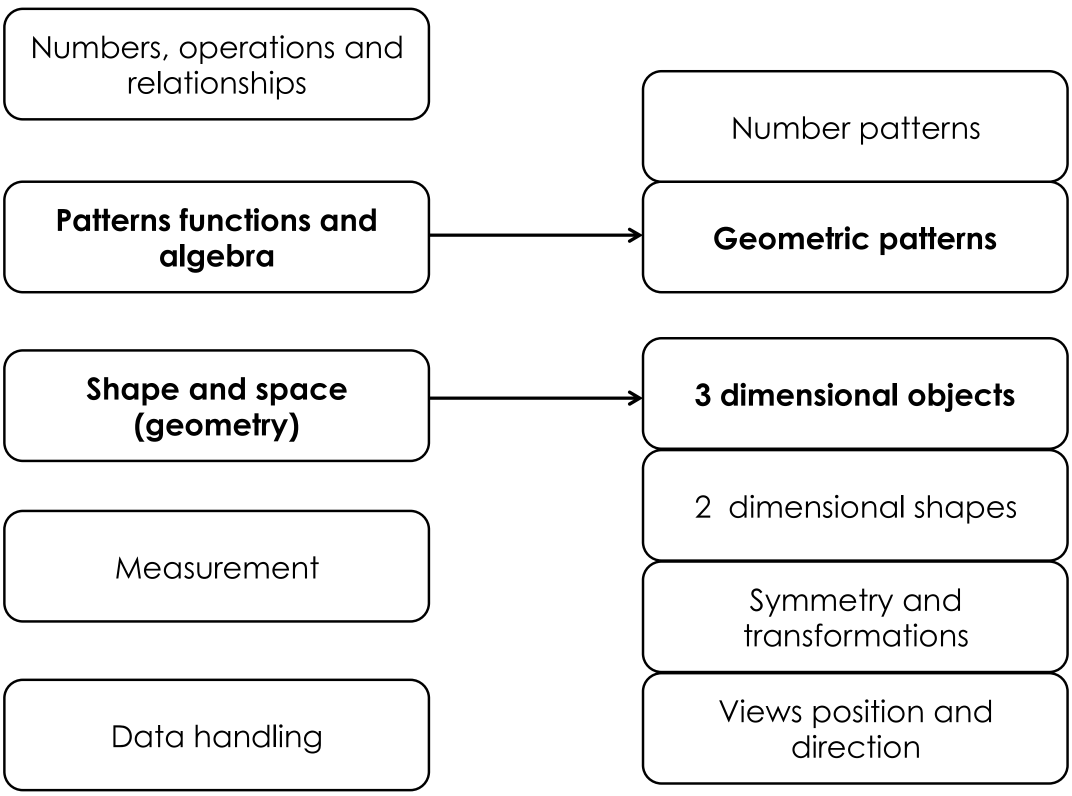
Contents

Introduction

Welcome to the Sci-Bono training workshop for lead teachers in Intermediate Phase Mathematics of the Gauteng Department of Education. This manual is a guide for training teachers in how to teach geometric patterns and properties of three-dimensional (3-D) objects in the Intermediate phase.

Training objectives

By the end of the training, participants should have embarked on a professional pathway towards ongoing reflective practice in mathematics teaching. There is a specific focus on geometric patterns and 3D objects. However, these topics fit into a bigger picture of mathematics. None of the topics can stand totally alone, and connection between topics and sub topics are very important.



During the 12-hour training workshop, participants will:

1. Solve problems about **geometric patterns**, that:
    - Connect number and geometric patterns and the underlying structures of all patterns;
    - Describe and extend patterns;
    - Distinguish growing from repeating patterns;
    - Identify the unit of repetition and
    - Define and use the 4 arithmetic operations or 4 geometric transformations to create patterns and/or find the rules underlying patterns;
2. Solve problems about **3D objects**, that:
    - Distinguish prisms from pyramids;
    - Break down 3D objects into their parts (2D surfaces, edges, vertices) and make 3D objects from these component parts);
    - Using 2D nets to create 3D objects;
    - Using dynamic geometry (GeoGebra) to explore properties 3D objects
3. Reflect on **pedagogical content knowledge** in terms of:
    - How mathematics is defined (and the connections between number work, patterns and geometry);
    - Current and preferred pedagogical approaches to teaching mathematics (for patterns and functions shape and space) in the Intermediate Phase; and
    - Questions that encourage mathematics thinking.

Workshop programme

	SATURDAY 6 MAY 2017
07:30 – 8:00	Arrival tea/coffee
08:00 – 9:00	Admin Activities & ice breaker
09:00 – 09:30	Pre-test
09:30 – 10:00	**TEA/COFFEE**
10:00 – 12:30	**Introduction** TASKS 1-4: Mathematics and connections between numbers and shape and space TASK 5: Reflecting on teaching practices
12:30 – 13:15	**LUNCH**
13:15 – 15:15	**Geometric Patterns** TASKS 6-9 (Task 8 is lesson planning, and could be shortened if time is a concern)
15: 15 – 15: 45	**TEA/COFFEE**
15: 45 – 17:15	TASKS 10-12: Using transformations to apply a rule and to create a geometric pattern

SUNDAY 7 MAY 2017

07:30 – 08:00	Arrival Tea/Coffee
08:00 – 10:15	TASKS 13-15: **3D Objects** Geometric solids, prisms and pyramids
10:15 – 10:45	**TEA/COFFEE**
10:45 – 12:15	TASKS 16-17: Exploring 3D objects, nets and dynamic geometry

12:15 – 12:30	Evaluation form completion
12:30 – 13:00	Post-test

Mathematical thinking

We all have slightly different understandings of what we mean by mathematics, and the mathematical thinking we use for shape and space (and the sub topics of 3D objects and geometric patterns).

Task 1: Mathematics

What is 'mathematics' all about? Please give a definition of mathematics.

What kind of processes are involved in mathematical thinking?

You should work on this, using:

THINK: work on your own first, to write down a definition);

PAIR: Share your definition with a partner. Try and merge and bring together your two definitions.

SHARE: Share your definition with the bigger group

Discussion guide

Many people – especially about children and parents - think that Mathematics is just about numbers and calculations. Sometimes when they include shape they focus on naming shapes – triangles, cylinders of pyramids. They don't think about change, movement, direction relating shapes to each other, construction or how numbers measurement and shape relate and connect with each other. Mathematics it is SO much more than numbers and calculations. It reveals hidden patterns that help us understand the world around us – according to Galileo, "Math is the language God used to write the universe". Mathematics is logical and involves us making sense of; inter alia, numerical and geometrical patterns. It is NOT just a series of rules to be applied blindly and mechanically, but also a series of concepts that can and should be understood.

Mathematics should be seen as a meaningful, sense-making activity where patterns and underlying structures are in focus. Children (and their teachers) should be engaged in a problem-solving process which involves them thinking deeply; reasoning and explaining their approaches to a community. Mathematics is not merely a process of mimicking already formulated procedural solutions.

Consult RESOURCE 1 for some further definitions of mathematics and the definition used in CAPS.

Patterns (connecting numbers and shapes)

Before we consider geometric patterns, we should first connect our ideas in shape and space, with ideas in number and pattern. The concepts we use in number patterns, can be applied to geometric patterns.

Number patterns

Task 2: Compare two number patterns

PATTERN A: 3; 6; 9; 3; 6; 9;… PATTERN B: 3; 6; 9; 12; 15;…

	Pattern A	**Pattern B**
Starts at		
Increases/decreases/ neither		
Grows/repeats		
How you would describe this pattern to someone else (so that they can replicate it)		
The 100th term is…		
Is 96 in the pattern?		
Justify your answer		

Discussion guide

When children/ teachers **describe** a growing number pattern they must pay attention to:

1. Where the pattern starts.
2. Whether the pattern increases (gets bigger) or decreases (gets smaller).

3. How to get from one number in the pattern to the next number. With numbers, they could use the four operations: add/subtract or multiply/divide. They could have a one step, or a two-step process.

However, when describing a repeating pattern, different language needs to be used. The description needs to include the fact that it is a repeating pattern, and the unit of repetition.

By intermediate phase, they also need to able to make a rule for a (growing) pattern. The rule relates the **position** (in the pattern) to the **value** (of the number). They can use this rule to

- predict which number will be in a certain position (eg which is the 10^{th} of 100^{th} number in the pattern?)
- find whether a particular number features in a pattern.

It can help to make a table or draw a function machine to find the rule.

Consult Resource 2 for a curriculum mapping of the key concepts and expectations per grade level relevant to shape and space. A full copy of the CAPS is available from www.education.gov.za

Task 3: Find a rule

PATTERN B is a growing pattern. It increases by the same amount each time.

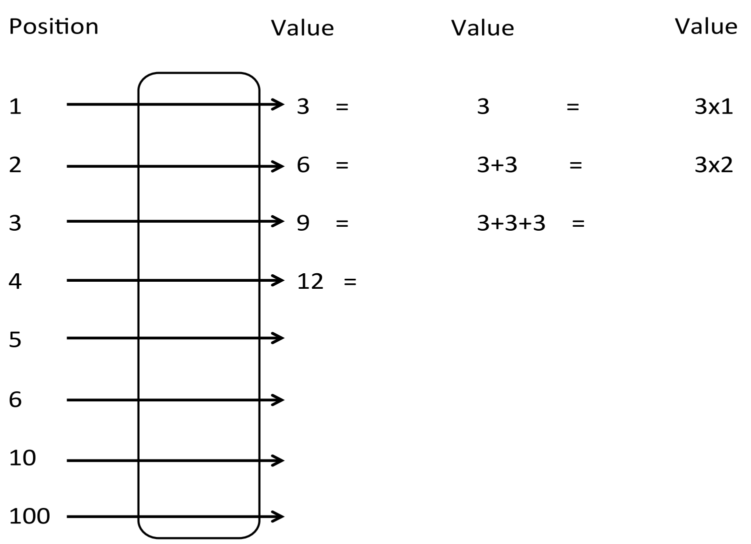
Find a rule for PATTERN B: 3; 6; 9; 12; 15; …

Use a table, or use a function machine.

TABLE

Position	**1st**	**2nd**	**3rd**	**4th**	**5th**	**6th**	**10th**	**100th**
Value	3	6	9	12				
Value	3	3+3	3+3+3					
Value	3 x 1	3 x 2	3 x 3					

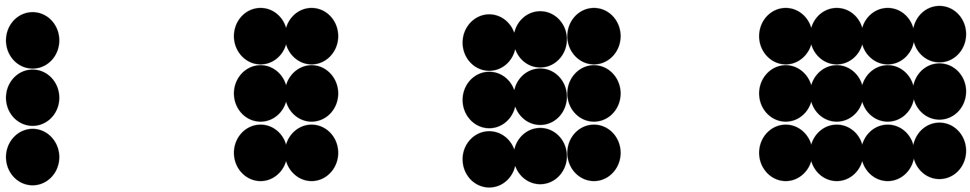
FUNCTION MACHINE



Discussion guide

When working with number patterns it is important to find the unit that is repeated. It may help to imagine or draw the numbers using shapes…

Pattern B



- What is staying the same, what is changing?
- What is the unit that is repeating each time?
- Is any part of the pattern growing or being added on each time?

When learners are finding a rule, it is important to give them time to work on investigating a solving the problem. It may be helpful to write down the gaps between each number in the pattern. If the gaps are same each time there is 'a common difference'. This action that is repeated is adding or subtracting the same amount each time.

There are many ways to find the rule, and different children will use different rules. The idea of equivalence (being the same value) is important. 3 + 3 + 3 is equivalent to 9 which is equivalent to 3 × 3. Realizing that repeatedly adding or repeatedly subtracting link to multiplying and dividing can be helpful.

Geometric patterns

The same ideas as above are used for geometric patterns. Geometric patterns can grow or repeat.

Consult Resource 2 for a curriculum mapping of the key concepts in patterns functions and algebra and for shape and space (geometry). Reflect on progression from one grade and one phase to the next.

Task 4: Compare two geometric patterns

Pattern A

Pattern B

	Pattern A	**Pattern B**
Starts at/with		
Grows/repeats		
The unit that repeats is…		
How you would describe this pattern to someone else (so that they can replicate it)		
Will the 96th shape be a circle or a triangle?		
Justify your answer		

True/False: The unit of repetition in both PATTERN A and PATTERN B is the same.

Discuss.

Discussion guide

When working with geometric patterns it is also important to find the unit that is repeated.

- What is staying the same, what is changing?
- Is there any part that is staying the same each time?
- Are there is any part that is growing in the same way each time?

Children may find thinking about a potato-print, or a stencil that has unit of repetition on it useful. They may circle or colour in or group the shapes which form the unit of repetition. Even in a growing pattern, there is often a base or basic unit which is being repeated. The unit of repetition then grows by the same amount each time.

Reflecting on teaching current practices

What is happening in your classroom? Who is doing the talking?

Task 5: Your teaching approach

1. Describe the kind of teaching approach that you are using to teach shape and space in mathematics.
2. What is the approach that you would advocate being used for shape and space?
3. What are the strengths and weaknesses of such an approach?

THINK – PAIR - SHARE

Discussion guide

Allow time to THINK (individually), PAIR (discuss in pairs) and then SHARE (making selections to the broader group). Participants will respond differently to this question. Allow each pair to thoroughly discuss their answers, and then encourage a few to share with the whole group. Allow others to provide comment/feedback on this input.

Try and shift the discussion from the general format of teaching approach (e.g. group-work versus chalk and talk) to focusing on their approach to teaching mathematics. What approach is adopted for the teaching of patterns and shape and space (practice makes perfect vs problem solving, vs realistic/ grounded mathematics or mathematics as presented rather than investigated). How is patterns approached? How is the similar or different to geometry?

There are some common approaches which can be used when teaching Mathematics. We use these ideas and approaches in the exemplar lessons which we work on later in the manual. They include:

- Mental maths – with ideas of mental activities involving shape and space
- Scaffolding in maths
- Problem solving approaches
- Using investigations
- Focusing on understanding and sense making rather than facts and procedures.

Consult Resource 3 for more detail on these ideas. Resource 3 offers some discussion about pedagogic approaches for mathematics teaching.

Have you had any discussion about **mental maths sessions** and how these apply in the mathematics classroom? How can we approach mental maths for geometric patterns and 3D shapes? Some suggestions:

1. Play 'Draw my Pattern'.

   The class closes their eyes and imagines a shape pattern. They listen to the description twice. Then they draw the pattern. For example:

      My pattern starts with a small circle.

      It is a repeating pattern.

The unit that repeats is circle, square, square.

The circle, and squares are all the same height.

The unit that repeats goes along a horizontal line.

Now draw my pattern.

Now someone else gives their description of the shape pattern…

2. Play 'I am holding a 3D object'.

   Work in pairs. Learners stand back to back. One learner is given a 3D object (box, cone, cylinder, cube, brick). That learner must describe the 3D object without naming it. They can use the words for 2D shapes. They can talk about faces, edges and vertices…The partner must imagine the object and say what they think it is and how big it is. For example:

   I am holding a 3D object. My object has 4 faces. All 4 faces are triangles. The edges are straight. The edges are all equal in length. My object fits in the palm of my hand.

There are more mental maths ideas in Resource 3.

Here are some aspects to reflect on:

- **Who does most of the talking throughout the lesson**? Do your lessons consist of mainly teacher talk or telling? In a problem-solving approach to mathematics classroom, 70% is learner talk and 30% teacher talk.
- **How is learning facilitated?** Do you explain how it needs to be done and make sure they understand it fully as possible before you allow them to work on their own? Do you allow create opportunities for learners to find their own solution paths or should they replicate your methods. When should they be replicating

your methods? Do you pull out learning from learners' mathematical thinking and use that to steer the rest of the lesson.

- **How confident are my learners to engage in non-routine problems?** Are learners confident to grapple with a problem? What do you do when they are not confident and reach a dead-end? What evidence are there that they are actually engaging with the problems?

Task 6: What type of questions do you ask?

We often spend time thinking and planning how we ask questions in test and exams. But we seldom think about what we ask and how we post questions during class time. Both types of questions – written and spoken - are important.

Spend some time now thinking about the questions you ask learners when you are talking to them in mathematics.

Write down at least 3 or 4 different questions that you ask learners questions during your shape and space maths lessons?

1, 2, 4, MORE

(1= think, 2=pair; 4= group of 4; MORE= share wider)

Discussion guide

Here are a few questions to reflect on:

- **What type of questions do I ask?** Do you ask questions that extend learners thinking, with follow up questions, questions requesting them to justify their answers, provide reasons or explain why they say so?

- **How well do I listen to learners' responses?** Do you listen with an evaluative ear? If someone gives an incorrect answer, you move to the following person to give the answer. Are you only interested in correct or incorrect responses or are you interested in the learners' mathematical reasoning, even if the response is incorrect.
- **Who answers the questions?** Is it mostly the same learners? Is it more the ones that can express them better? Is it often girls than boys?

There are three 'categories' of questions, questions asked throughout the lesson. Questions asked in the beginning of a lesson are different to those asked at the end of the lesson. The other category of questions is questions that stimulate different **levels of cognitive thinking**, for instance these questions are classified according to the Revised Cognitive Levels in Bloom's Taxonomy *(Remember, Understand, Applying, Analysing and Creating)*. This category of questions are familiar to teachers but at times, teachers only ask recall or facts questions and not higher cognitive demanding questions. The third category of questions are questions that encourage mathematical thinking. There are 6 aspects of mathematical thinking:

1. Exemplifying and specialising.
2. Completing, deleting and correcting.
3. Comparing, sorting and organising.
4. Changing, varying, reversing and altering.
5. Generalising and conjecturing.
6. Explaining, justifying, verifying, convincing, refuting**.**

There are questions that encourage these aspects of mathematical thinking. This category of questions will be unpacked throughout the lesson planning sessions.

- **Exemplifying, Specialising** Describe/demonstrate/show/choose/draw one of …
  Is there another? What's it like? Give me one/more examples of … Is … an example of …?
  What makes … an example? Can you find one that doesn't …? Are there any special ones?
- **Completing, Deleting, Correcting:** What must be added/removed/altered in order to allow/ensure/contradict …? What can be added/removed/altered without affecting …? Tell me what's wrong with … What needs to be changed so that…?
- **Comparing, Sorting, Organising:** What's the same about …? What's different about …? Sort or organise these by … Is it or is it not …?
- **Changing, Varying, Reversing, Altering:** What happens if we change …? What if …? If this is the answer to a similar question, what was the question?
  Do … in two or more ways. Which is the quickest/easiest …?
- **Generalising, Conjecturing:** Of what is this an example? What happens in general? Can you say why this is special? What happened here? And here? Can you see a pattern? Is it always, sometimes, never …? Describe all possible as succinctly as you can. What can change and what has to stay the same so that … is still true?
- **Explaining, Justifying, Verifying, Convincing, Refuting:** Explain why … Give a reason (using or not using…) How can we be sure that …? Tell me

what is wrong with … Is it ever false that …? (always true that …?) How is … used in …?

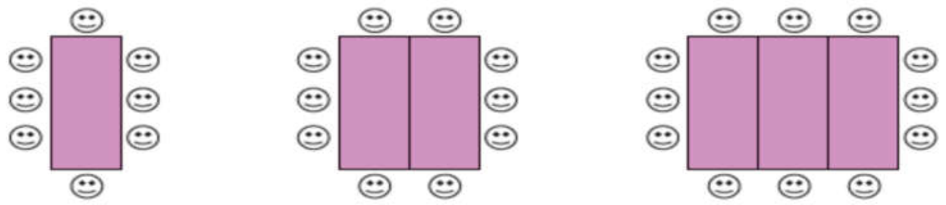
Explain the role/use of …

Solving problems about geometric patterns

In this section, we focus on geometric patterns and gives several investigations which you can work through, and then use with your class.

Task 7: Annah's Birthday party

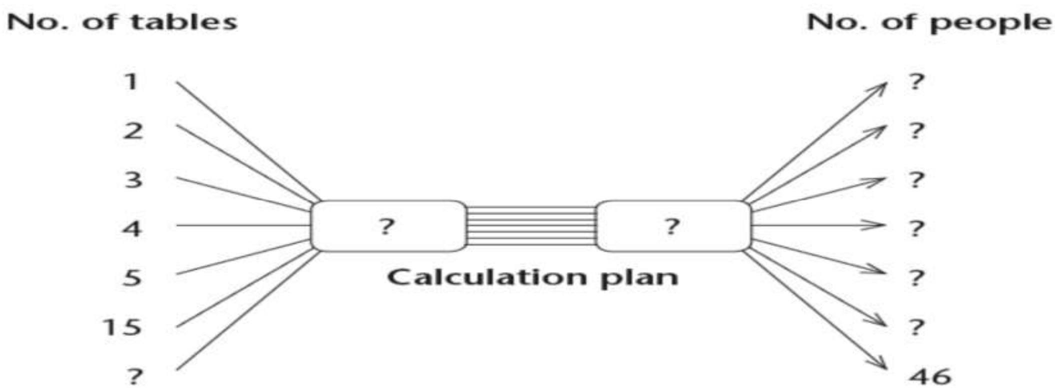
Annah is inviting many friend to her birthday party she thinks it would be better to seat her friend in the seating plan below.



1. Describe in words how the seating works.

2. With respect to the seating arrangement, what is staying the same what is changing? Is there a unit of repetition? What is growing?

3. Complete the flow diagram or function machine which relates the number of tables to the number of people.



4. Can you find a rule to relate the number of tables to the number of people?

If you don't like the flow diagram, you could make a table. You can also use the pictures of the tables and people to help you find the structure.

Discussion guide

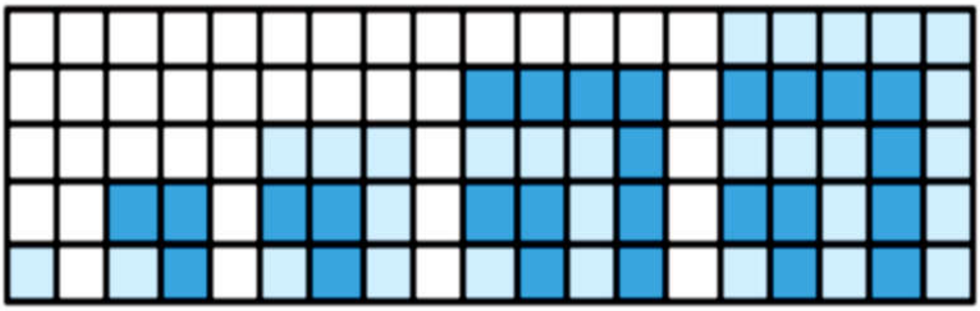
Allow time for discussing answers. Teachers should know the content they need to teach, the purpose of the activity is not to assess whether teachers are able to do the mathematics but to revise and summarise learners' conceptual development in Patterns, Functions and Algebra. Learners' conceptual development progress form recognition of patterns and relationships to the recognition of functions. Functions have unique output values mapped to a specific input value. Learners need to be able to investigate and extend the pattern, describe the general rules for the observed relationships and find input and output values. These concepts should be developed through mathematical thinking processes, especially generalising and conjecturing. The next section is an attempt to illustrate how learners can engage in mathematical processes, such as conjecturing, generalising and reasoning.

Lesson planning

Task 8: Planning a geometric patterns lesson

[*Note – Task 8 is quite long. If there is not enough time, this task should be omitted and returned to at the end of training, if there is time*]

In groups of four, plan a short lesson for Grade 6 which incorporates Sammy's pattern below as the main part of the lesson:



1. What will the lesson objective/s be?
2. Explain how and when you will use this diagram in a lesson.
3. Write down questions you will be asking when dealing with Sammy's pattern in the lesson

Discussion guide

The purpose for asking participants to plan short lessons is to gain an understanding of their thinking when planning lessons, especially how and when they will implement the diagram.

Each group should explain their short lesson plans to the rest of the group. Ideas might be similar, draw on those ideas, which are different. Questions could be asked for further elaboration from the group representative or the rest of the group. Pay attention to whether the lesson objectives speak to the concepts and skills needed to be developed and are the concepts and skills explicitly articulated.

The exemplar lesson plan (Resource 4 in the 'resource' document) is to illustrate how the geometric pattern can be used as an investigation. Trainers should read through the lesson plan with a focus on the main lesson drawing on the role of the teacher, specifically the questions asked and the learner activities throughout the lesson. Summary of learner activities encouraging mathematical thinking processes.

They should **describe** the pattern in words and extend the pattern.

Investigate the geometric pattern and **compare** to the geometric another geometric pattern.

Identify what is **similar** and what is **different** between the two patterns. Explain why they say it is different or similar.

Make **predictions or conjecture** about the 15th pattern based on what they observed. The teacher will ask learners to conjecture how many squares there are in the 15th pattern. Learners will explain their different predictions but the teacher does not affirm any predictions. Teacher explains that they need to investigate and prove that their conjectures are true.

Teacher explains what a conjecture is to guide learners in making conjectures and not just any guess.

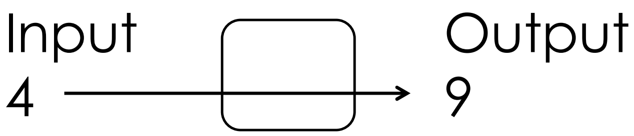
Teacher provides guidance by asking probing questions so learners can identify relationships between the pattern number and the number of odd numbers. NB! The teacher is not telling, but facilitating and guiding.

Task 9: Changing one number to another number

In numbers and number pattern work we spend a lot of time on the four operations (adding/subtracting, multiplying/dividing). These are the possible ways in which one number can change to another number.

Let's play I am thinking of a rule.

The input number is 4. The output number is 9. What is my rule?



1. Give me 3 different rules.
2. Make the hardest rule you can.

Discussion guide

There are many possible rules, such as:

- 'add 5'.
- - 'double and add 1'.
- 'multiply by 3 and then subtract 3': Start with 4.
- 'add $2\frac{1}{2}$, then add half of 10, then subtract half of 5'

Ideas for make a rule difficult could be to:

- use many steps in the rule
- to use different kinds of operations: adding/subtracting and multiplying/dividing

- use different kinds of numbers.

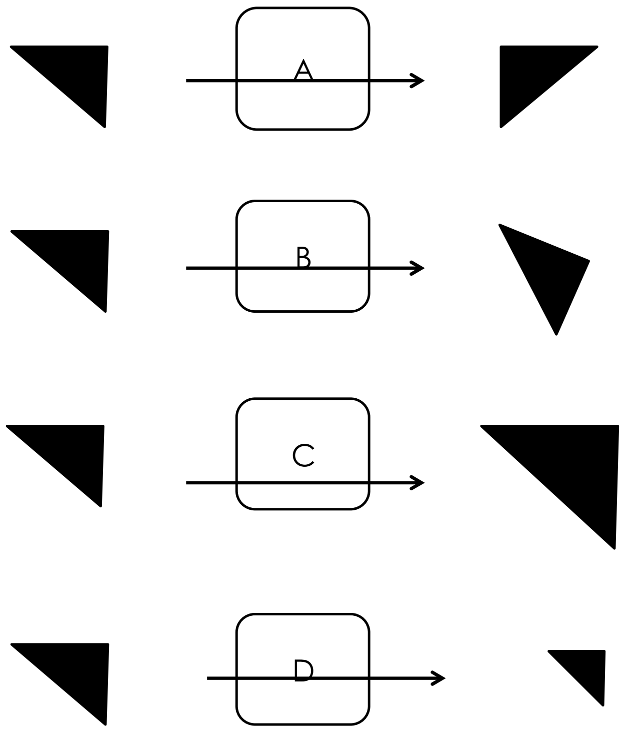
The key idea is that there are only four options for each step: adding/subtracting or multiplying or dividing. You can have many steps, but the four basic changes stay the same. If to change a number we can add or subtract, multiply or divide; and do that in many steps…What are the options for changing one shape into another shape.

Generating geometric patterns through geometric transformations

There are four transformations covered in the Intermediate Phase curriculum:

- translation
- reflection
- rotation, and
- enlargement

Task 10: Changing one shape into another shape



1. Look at how these triangles are changed. Which of the following have changed, and which stayed the same, in the case of each transformation:
   - Size
   - Shape
   - Orientation
2. The right-angled triangle is always the input. It goes into a machine A, or machine B or machine C or machine D and changes… What is the rule / the change / the transformation that happens in the machine?

Discussion guide

The important concept here is the idea of a change or a transformation. When we operate on a starting number (using the 4 operations) we change the input into a new output. We can work out the rule that is being applied to make the change.

The same idea holds for changing a geometric shape. There are four basic transformations:

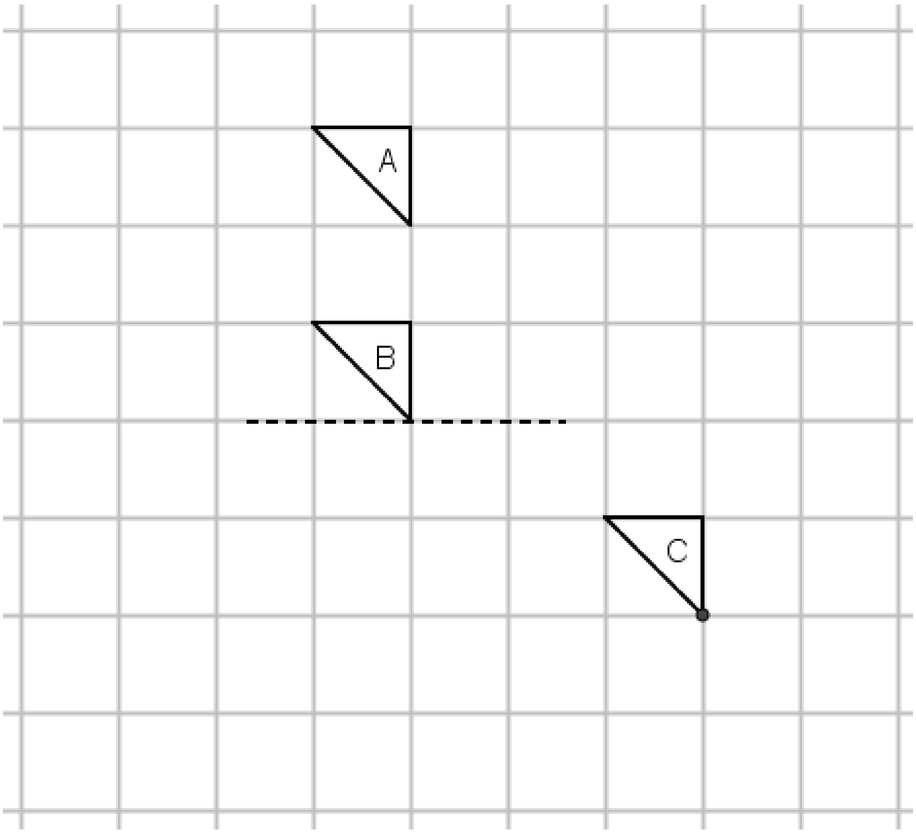
1. **Slide or translate.** This happens along a line, and has a direction (up, down, left or right). The slide/translation has a magnitude (how far you move the shape along the line). This movement can be measured using length (in mm, cm or m, or km).
2. **Turn or rotation**. This happens around a fixed point (the point of rotation) and also has a direction (clock wise or anti clockwise). This movement can be measured using angles (in degrees).
3. **Flip or reflect.** This is what happens to a shape based on a **mirror line** or **line of symmetry.**
4. **Enlarge or reduce.** This is when the shape stays in the same proportions, but every length is increased or decreased by the same amount (called the scale factor).

In the same ways as the four operations can be used to make one or two step rule for number patterns the four transformation can be used to create geometric patterns.

Task 11: Apply a rule or transformation

Draw in the new triangles on the grid below by following the transformations described

a. Translate triangle A two blocks to the right and 1 block up
b. Reflect triangle B in the dashed line
c. Rotate triangle C 90° anti-clockwise about the dot



Discussion guide

This example shows how to transform a right-angles triangle. Notice the detail of the rule:

1. There is an action: rotate, reflect, translate, enlarge

2. There is a direction: up, down, left, right, clockwise or anticlockwise
3. There is a magnitude: Move it by … blocks or length of… or angle of…

Can you apply the same rule again and again? So, each time you get a new shape, you apply the same rule again (repeat the rule). You can make a pattern.

Can you use the same rules, but have a different 2D shape or 3D object as the input? If the input was a rectangular prism (like a text book), what would the output be for each rule?

Task 12: Create a geometric pattern by applying a transformation

In each case, create a geometric pattern by following the given directions:

1. Reflect (flip) the triangle using a vertical line of symmetry that passes through the vertex that is furthest to the right, each time. Repeat until a geometric pattern is produced.

2. Translate the triangle 3 dots to the right, and then rotate it 90 degrees clockwise. Repeat!

3. Create three different examples of 2D shape patterns, that use one of the four transformations. For each pattern explain
   a. The unit of repetition
   b. The transformation used to get the next shape

Discussion guide

Compare shapes and objects in relation to other shapes and objects, you cannot compare differences and similarities if there is just one object or shape. You cannot generalise if there is only one shape. We can only see and describe similarities and differences in relation to others. Often teachers and children think that shape and space is static or still – it is just the naming of things, and interpreting pictures from books. But shape and space is dynamic – it is about how you transform or change one shape or object to another (in a new orientation, a new position or with new dimensions).

Three-dimensional objects

Imagine learners are asked to sort the following 3D objects:

A can of beans,
A toilet roll holder,
A pencil;
A drinking straw

There are different ways they could group them:

One group might put the straw and toilet roll holder together.

Why? Because they are both hollow tubes. The pencil and can go together as they are solid cylinders.

Another group of learners may sort these objects using their heights. First, they may orient all the shapes in the same way. They might lie the toilet tube and the can on their sides. In this way, the longest dimension (length) is now horizontal.

Alternatively, they could try and stand all the objects up – so the lengths are arranged vertically. Remember moving an object is transforming it. It is a common misconception that rotating an object will change the change object. Many children do not recognise a cylinder when it's height is not vertical.

Some children might order this using the length as the sorting attribute. Another way of sorting would be to sort from the smallest to the largest diameter. The vocabulary for 3D shapes becomes important in this discussion: orientation, base, circle, circular face, cylinder, tube, diameter, shortest tallest, longest, thinnest, vertical, horizontal, etc.

When learners sort, they may apply different 'rules of sorting'. Children should have to describe their sorting rule. They should identify which attribute they are focus on and how they are using this attribute to organise.

- Would your class recognise a 'shoe polish tin' as a cylinder?
- Would your class agree that a circle is a cylinder with zero or no height?
- Is a circle cut from a cereal box (cardboard) a cylinder?
- What about a paper circle? Is that a cylinder?
- What about an image of a circle shown on page or on a computer screen? Is that a cylinder?

Solving problems about 3D objects

Although 3D objects occur all around children, they often struggle to recognise this. In some curricula teachers start with the 3D objects, and then move to the 2D shapes which make up the 3D objects. Teachers often do not explore and make explicit the important attributes for each 3D object. They seldom show children examples of objects in the world which are not the classic 3D shapes.

Task 13: A cylinder or not a cylinder?

A group of Grade 5 learners was arguing whether the two objects in the pictures below are cylinders.

1. What do you think? Are these cylinders or not?

2. Explain what these learners should know about a cylinder to be able to know whether an object is a cylinder or not.

3. Check your vocabulary. A line can be straight or curved. A surface can be flat or curved.

A cylinder has _____ surfaces: 2 surfaces are ________ and 1 surface is _______.

Where two surfaces meet, we get a line, called call an ______________.

Where edges meet, we get a corner, called a __________________.

Discussion guide

Allow time for participants' responses and discussion. The purpose of this activity is to illustrate that children learn from both examples and counter examples. The point is if one knows cylinders then one will know what not cylinder is. There is also important vocabulary which should be used accurately in all discussions on shape and space.

Task 14: Comparing prisms and pyramids

Compare these two 3D objects:

What is the same and what is different?

You can work in pairs: one person is ROBOT (they can only write), the other person is PARROT (they can only speak and tell robot what to write).

Object A

Object B

Complete:

Both objects have the same base. The base is a regular ___________________.

Imagine making thin slices horizontally through each object:

For object A when you cut it into slices horizontally, you get regular ________. Each hexigon get smaller and smaller in size (similar). We call object A an hexagonal-based_______________________.

For object B when you cut it into slices horizontally, you get regular _______. Each hexigon is exactly the same size (congruent).

We call this an hexagonal- based _______________________.

Discussion guide

You can imagine that:

- A prism is a stack of regular shapes which are identical in size.
- A pyramid is a stack of regular shapes which get smaller and smaller (reduced in size) until it reaches a tiny point (the apex).

To name a prism or a pyramid you first need to decide on the base. That is the unit that is being repeated in slices. If each slice is exactly the same, then it is a prism. If each slice gets smaller and smaller, then it is a pyramid.

This child's stacking toy has a circle as its base.

Each ring in the stack gets smaller and smaller.

Is this toy like a circular based prism (cylinder) or a circular based pyramid (cone)?

Imagine that the toy had all the rings being identical in size (like the biggest ring on the bottom). If you stacked the identical rings, what 3d object would you get?

Task 15: Make a prism and a pyramid

Make rolled paper sticks. Work in a group of 4.

Your will need: sticky tape, paper, string and pair of scissors. Each person must make 5 rolled paper sticks.

You could also use: Straws and marshmallows or prestick. Or toothpicks and jelly tots

Roll up a piece or A4 paper. Put a piece of string that is longer than the paper, inside the roll. Use sticky tape to keep it tightly rolled.

As a group, use your rolled-up sticks to make:

1. A square-based prism
2. A square-based pyramid

Tie the strings/ or use the marshmallow/jelly tot to make each vertex.

	Square-based prism	**Square-based pyramid**
Number of edges (sticks)		
Number of vertices (corners)		
Number of triangular faces		
Number of square faces		

True or false: Every 3D shape has a pattern that repeats in each vertex. Discuss.

Discussion guide

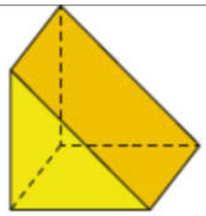
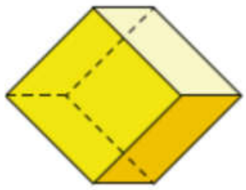
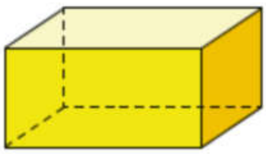
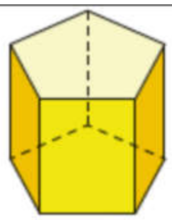
Use this activity to make sure that all participants can use this vocabulary correctly: edge, vertex, face, flat surface, curved surface, straight line, curved line. Can participants correctly identify and describe prisms and pyramids?

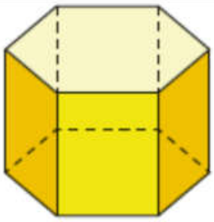
For groups that finish early you could challenge them to make other 3D objects. Use 2D surfaces that are either squares or equilateral triangles.

1. Make a shape that has 8 vertices. Every vertex is made the same way. There are always 3 squares that join to create each vertex. What is your shape?
2. Make a shape that has 6 vertices. Every vertex is made the same way. There are always 2 squares and 1 triangle that join to make each vertex. What is your shape?

Task 16: Exploring Euler's formula

Complete the following table:

3D object	Number of faces	Number of vertices	Number of edges	Name of 3D object
				
				
				
				

Check your work with a partner in a group. When you are sure you have all your values correct, answer these questions:

True or false: Vertices + Faces = Edges. Discuss.

Can you find a way to relate the vertices, faces and edges?

Discussion guide

Discuss if Euler's equation will hold true for all prisms and pyramids. The Euler equation is named after Leonard Euler, his equation was originally defined for polyhedra and used to prove various theorems of them and classifications of the platonic solids.

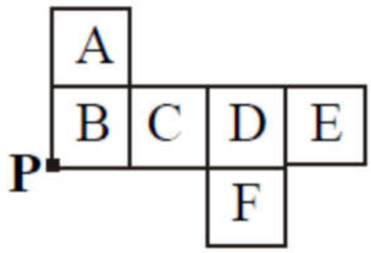
When working with 3D objects we can build them up from sticks (edges) and joins (vertices). We can also start with the 3D object and break them down to their component parts: surfaces or edges and vertices. In all cases, there is a repeating pattern which creates the polyhedron.

Another way of moving or changing or transforming from 2D to 3D and back again is to use nets.

Task 17: Nets

The following problem is one of the popular questions in a grade 5 Mathematics Challenge (Olympiad).

The net below must be folded to make a cube.



1. Which three faces will meet at P?
2. Discuss which activities do you recommend so learners would be able to identify which three faces will meet at P.

Discussion guide

Participants' response may vary from giving learners opportunities to fold nets, draw nets. Discuss: is there a particular pattern learners have to identify, working with 11 cube nets. Will they recommend the same activities for other 3-D objects? Allow participants to work with nets of different prisms and pyramids. Discuss the objects they made.

Dynamic Geometry

Many schools in Gauteng are getting access to ICTs. Dynamic geometry is a powerful tool which can be used to help children realise that shape and space – including geometric patterns and 3D objects is dynamic. There is movement like a video. Geometry is not static like a photo. In this final task, you can explore a simple investigation using a free dynamic geometry tool called GeoGebra.

Task 18: Dynamic geometry

Go to www.geogebra.org

You may have to create a free account (you can log in using Facebook)

Use and explore at least one of the following GeoGebra applications.

1. Nets for cubes: https://www.GeoGebra.org/m/C99EK3Dk
2. 2D shapes, 3D objects and nets: https://www.GeoGebra.org/m/sfXx3VzF
3. Exploring nets of geometric solids: https://www.GeoGebra.org/m/n6EjQDw8#material/NDPrtnEV
4. Enlargement: https://www.GeoGebra.org/m/bQ8xwXPx (explore scale factor and centre of enlargement)
5. Triangle: https://www.GeoGebra.org/m/tHYEyjgE
6. Area concept: https://www.GeoGebra.org/m/QAvaqhWU

You can search for existing materials and app in the search field.

Discussion guide

This is an exploratory task. Whenever a group is complete and waiting for other to complete, they could be going this website and exploring what they can find to help their teaching of geometric patterns and 3D objects. This can be uses a demonstration, or by children. There is a mobile application version where children can build shapes and objects using this software.